Focus Group Composition in Heterogeneous Populations

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Abstract

Often, groups of people behave differently than individuals. Influential members in a group can cause other members in the group to make irrational decisions. Focus groups are used within marketing to see how a group will react to a new idea or product. Focus group researchers will often create separate focus groups consisting of people who are alike so that influential members are less likely to occur. However, influence occurring in the population due to interactions consisting of heterogeneous people may not show up in these focus groups due to their homogeneity. In this paper I present a model of a population with two types of reactions to a new policy, non-dissenter and dissenter. Dissenters are against the new policy, and feel very strongly about their opinion. Non-dissenters are not against the policy, but their opinion is weak. If the policy is enacted, members of the population interact on a one-on-one basis, sharing their opinion of the policy until the population has reached a consensus on the policy. Modeling shows that even a small proportion of dissenters can cause large shifts in the average opinion in the population overtime. I then present a model of a focus group where members within the focus group share their opinion to all other members. I find that focus groups that consist of members from the targeted population can be used to estimate the population consensus, and that these focus groups can be composed of randomly chosen members of the population. I then go on to show that homogenous focus groups, or focus groups consisting of all non-dissenters or all dissenters, create a bias if used in the estimation.

1 Introduction

Groups of people are not people. Terms such as “groupthink” and “herd behavior” describe the often irrational behaviors that groups succumb to when trying to make a decision jointly or as indviduals. For example, the Bay of Pigs Invasion during the Cuban Missile crisis involved a group of highly skilled military tacticians who began to value the group more than their individual opinions. Questionable assumptions about the military strength of Cuba quickly became unquestionable truths to the group planning the invasion (Janis 1982) [5].

Focus groups are used to see how a group of individuals in a targeted population will react to a new product or idea (Lindlof, Taylor 2002) [8]. In the past 30 years, the use of focus groups in research has seen a marked increase. In 1977, only a couple dozen focus group reports were written, most of which were in market research. In 2001, several thousand articles were written in a variety of topics (Fern 1-4) [3]. This increase, however, has been met with criticisms. In this paper, I will focus on a particular criticism; Focus groups are prone to common “group biases” such as herd behavior, groupthink,and social desirability bias (MacDougall, Baum 1997) [9](Carey, Smith 1994) [1].

Focus groups tend to have six to eight members and someone to guide the flow of discussion, called the focus group moderator (Krueger 4) [7]. The moderator is responsible for asking the group questions and making sure everyone has equal weight in the discussion. The focus group moderator then takes notes or records audio and/or video while the focus group discusses the topic on hand (Krueger 137) [7]. This information is then handed off to the researchers, who look for similarities or dissimilarities between focus groups (Krueger 141) [7].

An advantage of focus groups is that as people in the focus group interact, there is a "chain" of ideas, and that members in the group feed off the ideas of other members in the group (Lindlof, Taylor 2002) [8]. However, this "chain" of ideas has often been criticized for being analogous to herd behavior or groupthink, and that individual opinions are not fairly considered (Rushkoff 2005) [11]. For example, if the focus group contains both males and females, focus group researchers note that males will often have stronger opinions, be more likely to convince the females in the group of their opinion, and are less likely to be swayed by the opinions of others in the focus group (Krueger 73) [7]. Another example of "groupthink" in focus groups is that people of different ages are usually separated in focus groups, as older members are often more persuasive than younger members (Fern 35) [3]. In general, people who traditionally have power or prestige in a society are more likely to convince those with less power or prestige and are less likely to be convinced by the opinions of others (Fern 30-32) [3]. This is problematic for focus group researchers because this means that certain opinions are censored or left out of the discussion (Krueger 72) [7].

The precautionary solution to this problem is to create homogenous focus groups (Krueger 72) [7]. For example, instead of having a heterogeneous focus group containing both men and women, have one focus group of just men and another with just women (Fern 35) [3]. Often this still does not solve the problem of "dominant talkers" (Krueger 111) [7] who still sway opinions within the group. Focus group researchers rely on the focus group moderator to keep these people from swaying opinions. Another solutions is to include a "devil's advocate" that will challenge dominant talkers with questions about their positions (MacDougall, Baum 1997) [9].

However, people are influenced by the opinions of others everyday. For example, women tend to speak less when there are more men and room as well as agree with their solutions to problems (Karpowitz, Mendelberg, Shaker 2012) [6]. Group communication outside of focus group research has no moderators, and those with weak opinions are quickly adopt opinions similar to their community. An example of this is New Coke: While it was popular in the initial market research phase, with New Coke having up to a 10-20% margin over Coca-Cola in blind taste tests (Pendergrast 352) [10], the product failed in less than eighty days after release. (Greising 113) [4]. When New Coke held focus groups, they decided to ignore the results because people would herd around those who were against the change (Greising 114) [4]. But it was exactly those who were against the change that would initiate the huge public backlash against New Coke. While some think New Coke showed that focus groups are useless(Rushkoff 2005) [11], others think we should shift the focus of focus groups: Focus groups can be used to estimate the effect of social influence (Schindler 1992) [12].

In this paper, I start with a principal trying to decide whether or not to enact a change of policy; For example, whether or not to change Coca-Cola with New Coke. The success of the policy is based on the average opinion in a population.The population consists of two types: dissenter and non-dissenter. When the non-dissenters hear the news about the change, they react positively, but do not feel strongly about the change. The dissenters, on the other hand, react negatively and feel very strong about the change. If the change is enacted, the dissenters will convince the non-dissenters that the change is a bad idea, lowering the average population opinion of the change over time. If the principal is aware of this influence of non-dissenters by dissenters, how will she predict the average opinion change?

The principal is aware of this, so she constructs focus groups which consist of both heterogeneous and homogenous mixtures of dissenters and non-dissenters. She then weights each focus group consensus about the policy enaction in order to create an estimator of the population consensus of the policy enaction. I find that as the focus groups grow in size and in number, the better this estimator is of the actual population consensus.

I then show that if the principal can not differentiate between non-dissenters and dissenters, then randomly sampling members of the population and putting them into groups can still estimate the population consensus.

Finally, I show that if the homogenous focus groups (those containing all non-dissenters or all dissenters) are removed and the estimator just uses heterogeneous focus groups to estimate the population consensus, not only will this new heterogeneous estimator also approach the population consensus, but for sufficiently large enough focus groups, the heterogeneous focus group estimator is better than the estimator that contains both heterogeneous and homogenous groups.

This would suggest that in populations with heterogeneous opinions and heterogeneous opinion strengths, heterogeneous focus groups may provide a better a prediction. Heterogeneous focus groups provide the advantage of seeing if one particular group can be influenced by another group.

2 Population Model

Suppose a principal is trying to decide if a policy will be successful or not. She has two choices: either to enact the policy or to not enact the policy. If she chooses to not enact the policy, nothing happens. Suppose the success of the policy depends of the average opinion of a population, which I will denote with E_t.

Let the population be arbitrary large and be composed of two types: non-dissenters and dissenters. Let p be the proportion of non-dissenters in the population, and let $(1-p)$ be the proportion of dissenters in the population. If a signal is sent from the principal to either the population or a particular focus group that the policy will be enacted, then non-dissenters set their prior distributions of their opinion of the policy to $\beta_{i,0} \sim N(1, \sigma_\beta^2)$ and dissenters set their prior distributions of their opinion of the policy to $\alpha_{i,0} \sim N(0, \sigma_\alpha^2)$ where $\sigma_\beta^2 \geq \sigma_\alpha^2$. In other words, the non-dissenter is generally positive about the policy change, but has a wider range of possible values for the policy change. The dissenters, on the other hand, have a much more narrow distribution of possible values for the policy change, and these values tend to be less than the non-dissenter. After the signal has been sent and priors have been chosen, the members of either the focus group or the population interact by sending signals to each other. Both non-dissenters and dissenters have normal likelihood distributions with $\sigma^2 = 1$.[1] If we allow $b = \frac{1}{\sigma_\beta^2}$ and $a = \frac{1}{\sigma_\alpha^2}$[2] then the updating equation for the any non-dissenter k will be[3]:

[1]See Appendix A.2 for further details

[2]These would be considered the precisions of the distributions for both the non-dissenter and dissenter, respectively.

[3]See Appendix A.1 for the derivation from a standard normal-normal conjugate

$$\beta_{k,m+1} = \frac{\beta_{k,t}(b+t)+x_t}{b+t+1}$$

and for any dissenter l:

$$\alpha_{l,m+1} = \frac{\alpha_{l,t}(a+t)+x_t}{a+t+1}$$

where is x_t is the signal of an opinion received at time t. Both of these updating equations represent a weighted prior opinion and signal of another opinion weighted by one. When either a non-dissenter or a dissenter receive a signal of an opinion, they form new prior opinions based on the opinion they received from the member they met and their former opinion[4]. Their former opinion is weighted based on their opinion strength, so that dissenters weigh their opinion more at any time period t. As t increases, both non-dissenters and dissenters weigh their opinion more and more.

2.1 Population Behavior

Suppose the principal enacts the policy and all members of the population receive the signal of the policy enaction. Once priors are chosen, population members interact by randomly pairing up and sending their mean value of their distribution of opinions. Let $\gamma_{i,t}$ be the mean of an arbitrary member of the population, where $\gamma_{i,t} \in \{\alpha_{i,t}, \beta_{i,t}\}$. Any consumer i is just as likely to meet any consumer j (uniform probability). Suppose at time period t consumer i meets consumers j, and they exchange opinions about the policy

[4]The reason this is assumed is because they could also "wait" for more opinions before they change their mind. This would result in the population eventually converging to p. However, people often do not "wait" for other opinions to determine their own. People will eventually give into social pressure, as having a certain opinion yields social capital vs being indecisive. For a series of experiments on this, see (Asch 1955) [2]

enaction. After they talk, both consumer j and i form new priors based on the opinion they received, in other words:

$$\gamma_{i,t+1} = \tfrac{\gamma_{i,t}(c_i+t)+\gamma_{j,t}}{c_i+t+1} \quad \text{and} \quad \gamma_{j,t+1} = \tfrac{\gamma_{j,t}(c_j+t)+\gamma_{i,t}}{c_j+t+1}$$

where $c_i, c_j \in \{a, b\}$. Since the proportion of dissenters and non-dissenters are $(1-p)$ and p, respectively, then the expected value function of the population at time t (or the average population opinion at time t) can be defined as:

$$E_t = pB_t + (1-p)A_t \tag{1}$$

where A_t is the expected value of all the dissenters, or

$$A_{t+1} = \frac{A_t(a+t) + E_t}{a+t+1} \tag{2}$$

and B_t is the expected value of all the non-dissenters, or

$$B_{t+1} = \frac{B_t(b+t) + E_t}{b+t+1} \tag{3}$$

Both B_t and A_t can be seen as what happens to the average non-dissenter or dissenter at time t (respectively). At time 0, $B_0 = 1$, $A_0 = 0$, and $E_0 = p$. Given enough interactions in the population, the opinions of the dissenters and the non-dissenters become more and more alike, until they eventually reach a consensus.

Definition 1 *A consensus, $\mu_f(a, b, p)$, is when the limits expected value of both the non-dissenters and dissenters have the same mean, or,* $\lim_t(A_t) =$

$lim_t(B_t) = lim_t(E_t) = \mu_f(a, b, p)$

Theorem 1 *The population reaches a consensus as t goes to infinity*

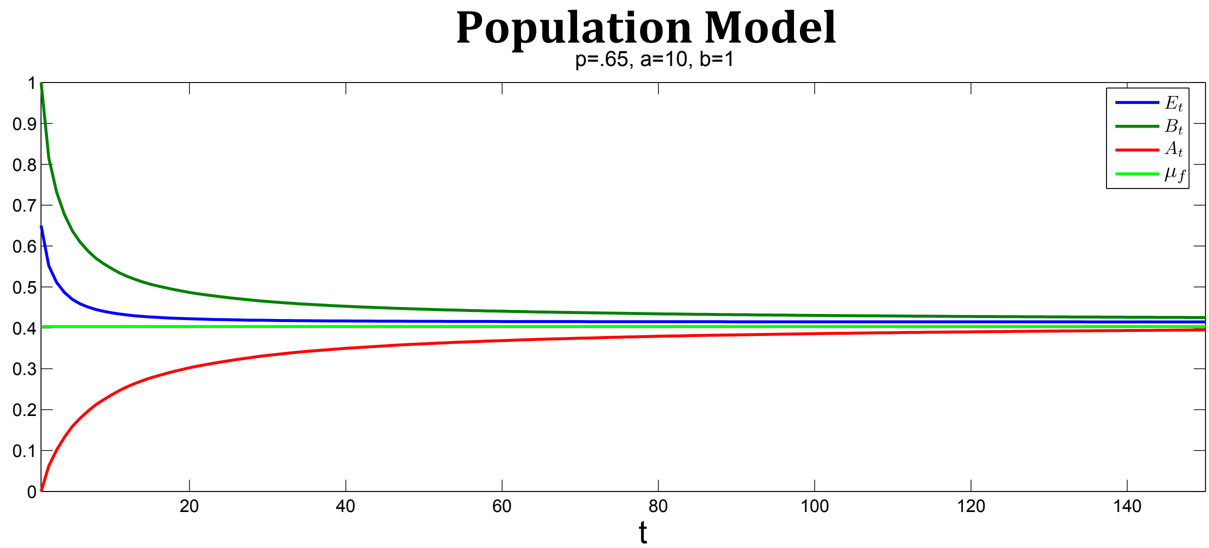
Proof:

Suppose $A_{t+1} = A_t$. Then:

$$A_t = \tfrac{A_t(a+t)+E_t}{a+t+1}$$
$$A_t(a+t+1) = A_t(a+t) + E_t$$
$$A_t(a+t+1-(a+t)) = E_t$$
$$A_t = E_t$$

A similar proof can be done for the sequence B_t. Thus when $A_t = A_{t+1}$, $A_t = E_t$ and when $B_t = B_{t+1}$, $B_t = E_t$. By Corollaries 1.1, If $A_t \neq A_{t+1}$, then $A_t < A_{t+1}$. Similarly, by Corollaries 1.2, if $B_t \neq B_{t+1}$, then $B_t > B_{t+1}$. By extension of corollary 1.3, if $A_t \neq E_t \neq B_t$, then $B_t > E_t > A_t$. Finally, by Lemma 1, since $B_t \geq A_t$, then the a consensus is reached when $A_t = B_t = E_t$.

This behavior can be seen in the graph below:



2.2 The Naive Prediction

Definition 2 *The **naive prediction** is the principal's prediction when she assumes that dissenters and non-dissenters will not influence each other*

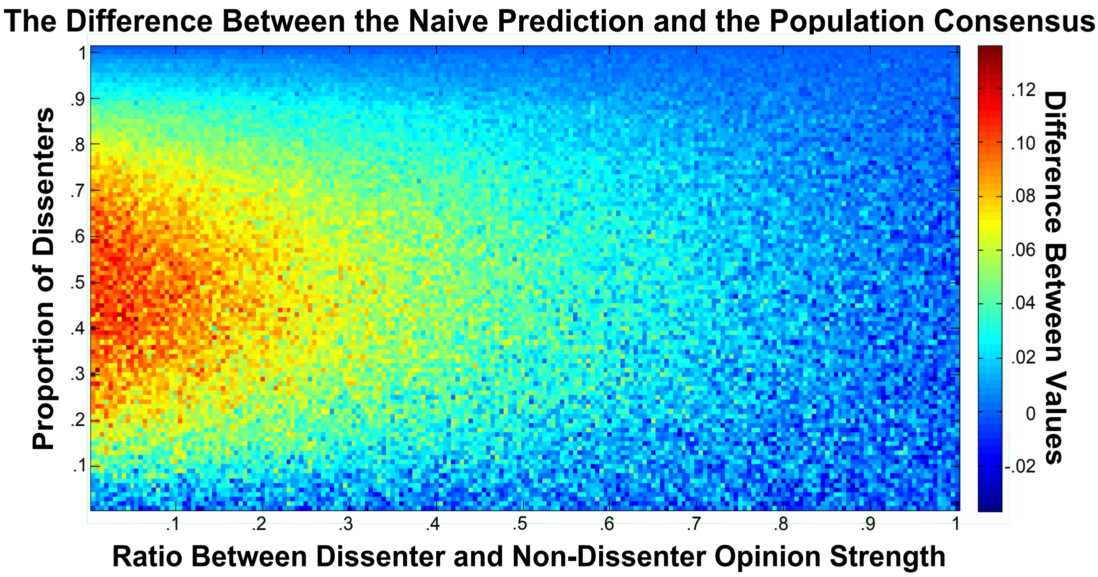
In other words, naive prediction is the belief that all non-dissenters will support the policy enaction and the dissenters will not. If she were to be "naive", she would believe that the opinions of neither the dissenter nor the non-dissenter will change when they interact. If we interpret the means of both dissenters and non-dissenters as the probability that will support the enaction, then the average naive prediction per member is p. I will later show that using only homogenous focus groups will yield the same result.

The naive prediction can sometimes be correct, if we difference the expected value function once, then:

$$\Delta E_t = E_{t+1} - E_t = p(1-p)(A_t - B_t)\frac{a-b}{(1+b+t)(1+a+t)}$$

If $a = b$, $p = 0$, or $(1-p) = 0$ then $\Delta E_t = 0$, and $E_0 = \mu_f = p$. In other words, if the population has homogenous opinions (all non-dissenters or all dissenters) or opinion strength, then the population consensus is the prior population average opinion, or p.

However, if $a \neq b$ and $0 < p < 1$, then the naive prediction will be wrong. Below is a graph of the difference between naive prediction and the population consensus, or $p - \mu_f(p, a, b)$:



3 Focus Group Model

Suppose that the principal knows p, the proportion of non-dissenters in the population, and that she can differentiate between the two types in the population (dissenter and non-dissenter)[5]. Assume that the principal can not

[5] This assumption is removed in Section 4

determine a, b, and the functional form of $\alpha_{i,t}$ or $\beta_{i,t}$. In order to estimate the population consensus, the principal will create focus groups of size n.

Definition 3 *A* ***focus group*** *is a tuple of non-dissenters and dissenters*

Let β_i denote a non-dissenter in the ith position in the focus group, and let α_j denote a dissenter in the jth position in the focus group. The set of focus groups of size n, or Ω_n is Cartesian product of $\{\beta_i, \alpha_j\}^n$, or for any size n, $\Omega_n = \{\beta, \alpha\}^n$. Thus, for any n, the number of possible focus groups is 2^n. Let $X_i^n \in \Omega_n$ denote a particular focus group.

The principal creates the focus group X_i^n by picking out n consumers and ordering them in a set so that $X_i^n = (\gamma_1, \gamma_2, \dots \gamma_n)$ where each $\gamma_i \in \{\alpha_i, \beta_i\}$, and has an opinion strength $c_i \in \{a, b\}$. In other words, a ordered list of dissenters and non-dissenters. For example, the focus group $(\beta_1, \alpha_2, \beta_3)$ denotes a focus group of size three with a non-dissenter in the first position, a dissenter in the second position, and another non-dissenter in the third position. The focus group starts by the moderator allowing γ_1 to send his signal to $\gamma_2, \dots \gamma_n$, after which all but γ_1 updates and forms a new prior mean. For example, in the focus group $\{\beta_1, \alpha_2, \beta_3\}$, β_1 will speak. Thus, for any consumer $i \neq 1$ in the focus group, the distribution of their mean can be written as followed:

$$\gamma_{i,1} = \frac{\gamma_{i,0}(c) + \gamma_{1,0}}{c_i + 1} \tag{4}$$

The moderator then allows γ_2 to speak, thus for any consumer $i \neq 1, 2$

$$\gamma_{i,2} = \frac{\gamma_{i,1}(c+1) + \gamma_{2,1}}{c_i + 2} \tag{5}$$

Which can be reduced to

$$\gamma_{i,2} = \frac{\gamma_{i,0}(c) + \gamma_{1,0} + \gamma_{2,1}}{c_i + 2} \tag{6}$$

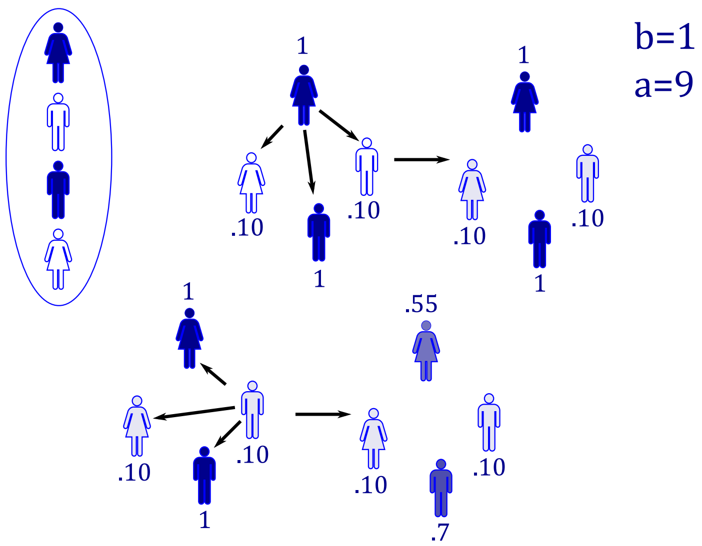
This process will continue, until the *ith* step, where the *ith* member signals everyone (but do not signal themselves). In general, for any consumer i, once every consumer has given their signal, can be written as followed

$$\gamma_{i,n-1} = \frac{\gamma_{i,0}(c) + \sum_{j=1,j\neq i}^{n} \gamma_{j,j-1}}{c_i + n - 1} \tag{7}$$

Define a "round" as going through all consumers once, so that at any round[6] u, $r_u = u(n-1)$. As more and more rounds occur, the opinions of the non-dissenters and dissenters become closer and closer together, and eventually a consensus will occur among the focus group members.

Below is an example of the focus group $(\beta_1, \alpha_2, \beta_3, \alpha_4)$ interacting and updating, where b=1 and a=9:

[6] An alternative way of writing these equations with the mean instead of the summation of all other players is shown in Appendix C



3.0.1 Heterogeneous and Homogenous Focus Groups

Definition 4 *A* ***homogenous focus group*** *is a focus group which contains all non-dissenters or all dissenters*

Definition 5 *A* ***heterogeneous focus group*** *is a focus group which is not a homogenous focus group*

For any set of focus groups of size n, Ω_n, there is exactly two homogenous focus groups: the all non-dissenter focus group, which I will denote by X_β, and the all dissenter focus group, which will be denoted by X_α. Since for any n both of these groups all have the same prior and all send the same signal their consensuses will always be $x_\beta = 1$ and $x_\alpha = 0$.

Definition 6 *The* ***naive focus group estimator***, ϕ_n, *is a focus group estimator which uses only homogenous focus groups*

Suppose we choose both homogenous focus groups of size n in the model and weighted them accordingly, or:

$$\phi_n = pX_\beta + (1-p)X_\alpha$$
$$\phi_n = p(1) + (1-p)(0)$$
$$\phi_n = p$$

Notice that this is the same as the "naive prediction", or the prediction that the principal believe she will earn if she thought that the dissenters would not influence the non-dissenters.

In Section 4, I will show that there is a systematic bias from homogenous focus groups and that removing both of the homogenous focus groups provides a better estimate when there is a large difference between the opinion strengths, b and a, of the non-dissenters and the dissenters.

Theorem 2 *Any focus group X_i^n reaches a consensus, $lim_u(X_i^n) = x_i^n$*

Proof:

This follows from Lemma 2 and Corollary 3.1, if there is a unique dissenter, α_i, and a unique non-dissenter, β_j, so that for any round r_u $min(X_{i,n}) = \alpha_i$ and $max(X_{i,n}) = \beta_j$ and $\alpha_{i,r_u} < \alpha_{i,r_{u+1}}$ and $\beta_{j,r_u} < \beta_{j,r_{u+1}}$ as long as $\alpha_{i,r_u} \neq \beta_{j,r_u}$.

In a similar proof to Theorem 1, when $\alpha_{i,r_u} = \alpha_{i,r_{u+1}}$, then α_{i,r_u} is equal to the average opinion value of the focus group at the end of a paticular round, $\hat{\gamma}_{r_u}$. Similarly, if $\beta_{j,r_u} = \beta_{j,r_{u+1}}$, then $\beta_{j,r_u} = \hat{\gamma}_{r_u}$. Since both the maximium and the mininium are equal to the mean, they must be equal to each other, or $\beta_{j,r_u} = \hat{\gamma}_{r_u} = \alpha_{i,r_u}$. Thus, every memeber of the focus group must be equal to each other. Therefore, if there were expected values for both the non-dissenters and dissenters, they would be equal as well.

The difference between Theorem 1 and Theorem 2 is that in the population, any member i can meet any member j. This random process creates systematic error similar to a random walk. Thus, proving that a consensus exists involves an expected value function, E_t. The focus groups, on the other hand, do not suffer from this type of error. Theorem 2 involves finding the unique maximum non-dissenter and the unique minimum dissenter. Then showing that after each round, the distribution of the minimum dissenter is monotonically increasing around the mean, while the distribution of the maximum non-dissenter is monotonically decreasing.

After all focus groups in the set of focus groups, Ω_n, have reached a consensus, the principal takes the weighted mean of the probability of picking out that ordering. For example, if $p = .6$, then the probability of picking out a non-dissenter, a dissenter, and a non-dissenter is $(.6)(.4)(.6) = .144$, thus the consensus of the focus group $\{\beta_1, \alpha_2, \beta_3\}$ will be weighted by .144. These weighted consensuses are added up together to create an estimator, ζ_n, where n is the size of the focus groups. Let $\mathbf{x_n}$ be the set of all focus group concensuses for focus groups of size n. So that for focus group i of size n, $x_i^n \in \mathbf{x_n}$. If p is the proportion of non-dissenters in the population, then ζ_n is calculated by"

$$\zeta_n = \sum_{j=0}^{n} \sum_{x_i^n \in \mathbf{x_n}} (p^{n-j})((1-p)^j)(x_i^n)$$

Theorem 3 *The limit of* $\zeta_n(p, a, b)$*, or* $lim_n(\zeta_n(p, a, b))$ *is the population consensus,* $\mu_f(p, a, b)$

Consider the distribution of opinion values at time period 0. At time period 0, signals are sent from the principal to the population. There are two opinion values possible, that of the non-dissenters, whose distribution is centered at the opinion value 1, or $\beta_{i,0} = 1$, and that of the dissenters, whose distribution is centered at the opinion value 0, or $\alpha_{j,0} = 0$. Thus, we can write a probability distribution of opinion values at time period zero:

$$\mathrm{P}_0(X = x) = \begin{cases} p & \text{if } \beta_0 \\ (1-p) & \text{if } \alpha_0 \end{cases}$$

At time period one, the distribution now has four different opinion values:

1. A non-dissenter met another non-dissenter with probability p^2
2. A dissenter met another dissenter with probability $(1-p)^2$
3. A non-dissenter met a dissenter with probability $p(1-p)$
4. A dissenter met a non-dissenter with probability $(1-p)p$

Thus, at time period 1, the probability distribution is:

$$\mathrm{P}_1(X = x) = \begin{cases} p^2 & \text{if } \beta_0(\beta_0) \\ (1-p)p & \text{if } \beta_0(\alpha_0) \\ (1-p)p & \text{if } \alpha_0(\beta_0) \\ (1-p)^2 & \text{if } \alpha_0(\alpha_0) \end{cases}$$

In general, at any time t, there are at most 2^{t+1} possible opinions to be held in the population. Suppose z_t^i is a particular opinion held at time period t and let $z_t^i \in Z_t$ denote the set of all opinion values at time t. Suppose an arbitrary opinion held at time, z_t^i, is the result of meeting $\hat{a}$ dissenters and $\hat{b}$ non-dissenters, where both $\hat{a}$ and $\hat{b}$ include the signal that the consumer received at time period 0. In other words, the consumer meets themselves at time period zero. The probability of a particular consumer having the opinion z_t^i is then $(p)^{\hat{b}}(1-p)^{\hat{a}}$. Let P_t be the probability distribution of all possible opinions at time period t, z_t^i. In general[7] :

$$P_t(Z_t = z_i^t|\hat{a}, \hat{b}) = p^{\hat{b}}(1-p)^{\hat{a}}$$

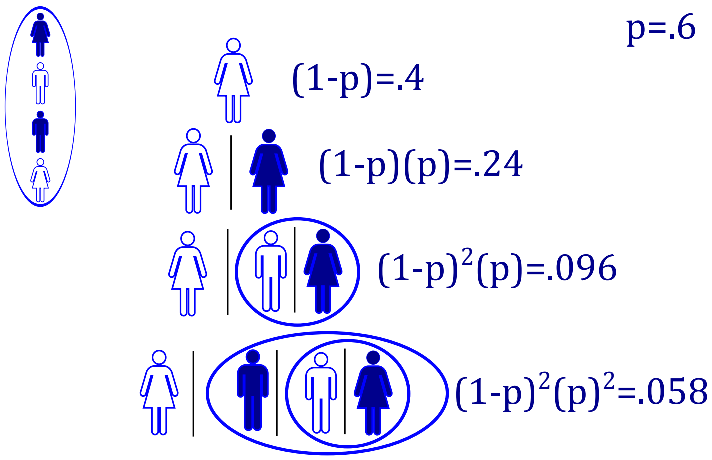
This allows us to write the expected opinion value at time period t, E_t, as:

$$E_t = \sum_{z_t^i \in Z_t}(z_i^t)[P_t(Y = z_i^t)]$$

Consider the last member to speak in any focus group of size n. For example, in the focus group $(\alpha_1, \beta_2, \alpha_3)$, then α_3 would be the last member to "speak" each round. Now consider the set of the opinion values of all last members to speak of focus groups of size n at the end of the first round. For example, after α_1, β_2,and α_3 speak, the opinion value of α_3 would be in this set. Call this set ω_n. Consider the case of ω_1, since both focus groups are of size one and are the homogenous focus groups X_β and X_α, $\omega_1 = 1, 0$. This is the same as Z_0, since this is when signals are received and the only values are 1 and 0. Below is an example of the relationship between opinion values during population interaction and the last member of each focus group. Specifically, P_3 and the focus group $(\beta_1, \alpha_2, \beta_3, \alpha_4)$. Notice that the last member in the focus group of size four would have a corresponding opinion value in the population at time period 3[8]:

[7]Visual examples of P_t for time periods $t = 0, 1$ and 2 are shown in Appendix D.1

[8]A more complete example for $n = 2$ and $t = 1$ is shown in Appendix D



In general:

$$\omega_n = Z_{n-1}$$

Now consider the set of all consensus values for all focus groups of size n, $\mathbf{x_n}$. If n increases without bound, then a focus group of size n will "almost surely" reach a consensus before the last member of each focus group of size n has spoken. When a focus group reaches a consensus, all members in the focus group have the same value x_n^i. Thus, the last member of each focus group of size n has an opinion value that is the consensus. Thus:

$$lim_n(\mathbf{x_n}) = lim_n(\omega_n)$$

Since the $\omega_n = Z_{n-1}$, then $lim_n(\mathbf{x_n}) = lim_n(\omega_n) = lim_n(Z_{n-1})$. From Theorem 1, the limit of the expected value of opinions in the population is the population consensus, or $lim_t(E_t) = \mu_f$. From above, we know that:

$$E_t = \sum_{z_t^i \in Z_t}(z_i^t)[P_t(Y = z_i^t)]$$
$$lim_t(E_t) = \sum_{z_t^i \in lim_t(Z_t)}(z_i^t)[P_t(Y = z_i^t)]$$

$$\mu_f = \sum_{z_t^i \in lim_t(Z_t)} (z_i^t)[P_t(Y = z_i^t)]$$

Since $lim_t(Z_t) = lim_n(\mathbf{x_n})$, we can replace all opinion values at time period t, z_i^t with focus group consensuses x_i^n. Thus:

$$\mu_f = \sum_{x_i^n \in lim_n(\mathbf{x_n})} (x_i^t)[P_n(Y = x_i^n)]$$

The probability distribution of the consensuses would be the weights attached to them[9]. For example, the probability of the principal of randomly picking out two non-dissenters and one dissenter is $p^2(1-p)$. Hence:

$$\mu_f = lim_n(\zeta_n)$$

Thus, the limit of the focus group estimator is the population consensus.

Theorem 4 *The focus group estimator $\zeta_n(p, a, b)$ is a strictly consistent estimator for the population consensus $\mu_f(p, a, b)$*

We wish to show that the focus group estimator is monotonically decreasing, or:

$$\zeta_1 \geq \zeta_2 \geq \cdots \geq \zeta_n \geq \zeta_{n+1} \tag{8}$$

From Lemma 6, if $a = b$, then $x_i^n = \bar{\gamma}_0$, this reduces ζ_2 to:

$$\zeta_2 = p^2 + p(1-p)x_1 + p(1-p)x_2$$
$$\zeta_2 = p^2 + p(1-p)(.5) + p(1-p)(.5)$$
$$\zeta_2 = p^2 + p(1-p)$$
$$\zeta_2 = p(p + 1 - p)$$
$$\zeta_2 = p$$

[9]This idea is further explained in Theorem 5

and in general

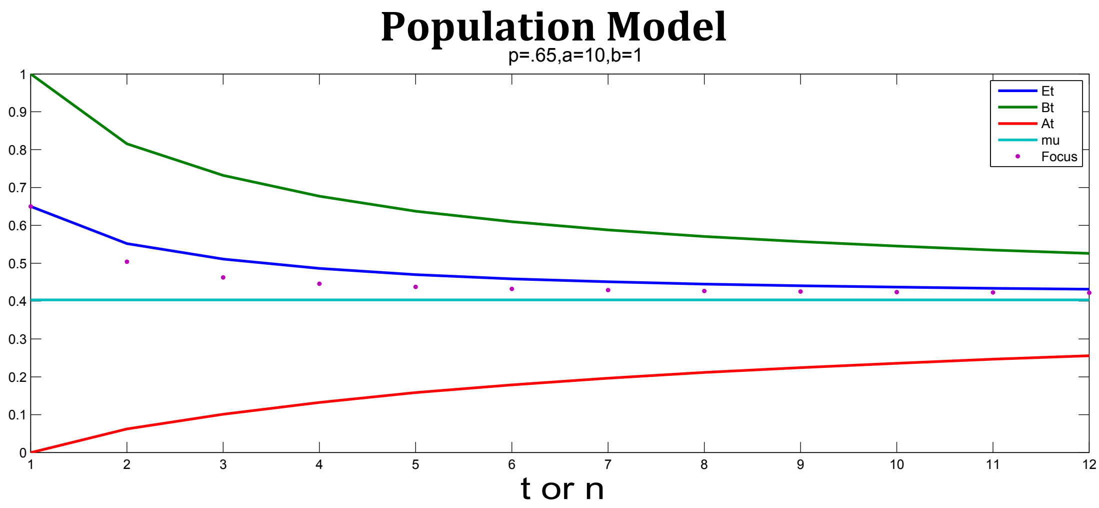
$$\zeta_n = p^n + p^{n-1}(1-p)(\tfrac{n-1}{n}) + \cdots + p(1-p)^{n-1}(\tfrac{1}{n})$$
$$\zeta_n = p^n + p^{n-1}(1-p) + \cdots + p(1-p)^{n-1}$$
$$\zeta_n = p(p^{n-1} + p^{n-2}(1-p) + \cdots + (1-p)^{n-1})$$
$$\zeta_n = p\textstyle\sum_{i=0}^{n-1} p^{n-1-i}(1-p)^i$$
$$\zeta_n = p\textstyle\sum_{k=0}^{n-1} p^{k-i}(1-p)^i$$
$$\zeta_n = p$$

Thus, if $a = b$, $\zeta_1 \geq \zeta_2 \geq \cdots \geq \zeta_n \geq \zeta_{n+1}$, since $p \geq p \geq p \cdots \geq p \geq p$.
If a goes to infinity, then every focus group consensus expect for the focus group containing all non-dissenters, X_β, will be zero, or for any $X_i^n \in \Omega_n$, $lim(X_i^n)_u = x_i^n = 0$. Thus, $\zeta_n = p^n$. Thus if a goes to infinity, $\zeta_1 \geq \zeta_2 \geq \cdots \geq \zeta_n \geq \zeta_{n+1}$ because $p \geq p^2 \geq \cdots \geq p^n \geq p^{n+1}$.
From Lemma 5, we showed that for every $x_i'(a) \leq 0$, thus for every $a \in [b, \infty)$:

$$\zeta_1 \geq \zeta_2 \geq \cdots \geq \zeta_n \geq \zeta_{n+1}$$

Thus, ζ_n is a strictly consistent estimator for $\mu_f(p, a, b)$.

Below is a graph illustrating Theorem 4, the purple dots represent ζ_n of size $n = 1, 2, \ldots, 12$ and the functions A_t, B_t, and E_t for time periods $t = 0, \ldots, 11$.



This means that if we increase the size and number of focus groups in a systematic way, the estimator ζ_n becomes consistently better. Suppose we decided to be systematic in a different way, for example we increase the size of the focus groups but screen who we choose to only have homogenous focus groups. If this were the case, the model would give us p every time, since this is the average between the homogenous non-dissenter group and the homogenous dissenter focus group, no matter how large n becomes.

In the next section, I will show that you can drop the systematic screening of non-dissenters and dissenters in the model and still approach ζ_n and that this estimator is better than the all homogenous focus group estimator.

4 Removing the Differentiation Assumption

At the beginning of the focus group model section, the assumption was made that the principal knew p and could differentiate between the non-dissenters and dissenters. In this section, I will show that both of these assumptions can be removed, and we can still approach the estimator ζ_n. For example, suppose the principal chose to use focus groups of size three, or $n = 3$. The principal randomly chooses a consumer γ_1 from the population, and puts this consumer 1^{st} in an ordered set. She also picks a second and third consumer, γ_2, γ_3, who will be the 2^{nd} and 3^{rd} members, respectively. Suppose the proportion of dissenters in the population is p. Then, with probability p^3, this set will be $\{\beta_1, \beta_2, \beta_3\}$. With probability $p^2(1-p)$, the set will be $\{\beta_1, \alpha_2, \beta_3\}$. In general, the probability of sampling n individuals and putting them in a particular focus group with **b** non-dissenters and **a** dissenters is $p^{\mathbf{b}}(1-p)^{\mathbf{a}}$. Since we are sampling focus groups $X_1^n, X_2^n, \ldots, X_{2^n}^n$ with the probability distribution of their weights, we will get the same exact result as above. Define $\hat{\zeta}_n$ as the sample mean of the focus groups. In other words, if we sample v consumers in put them into focus groups of size n and $\frac{v}{n} = h$, where h is a positive integer, and each focus group consensus is denoted by y_i^n then:

$$\hat{\zeta}_n = \frac{1}{h} \sum_{i=1}^{h} y_i^n$$

4.0.2 Theorem 5

$\hat{\zeta}_n$ is an unbiased estimator for the estimator ζ_n

Proof:

We wish to show that $E(\hat{\zeta}_n) - \zeta_n = 0$. Suppose h focus groups are sampled and, as in Section 3, reach a consensus y_i , and let the sample set of focus group consensuses be $y_1^n, y_2^n, \ldots, y_h^n$. Since:

$$\hat{\zeta}_n = \frac{1}{h}\sum_{i=1}^{h} y_i^n$$

group all $y_i^n = y_j^n$ so that we can rewrite the above equation as

$$\hat{\zeta}_n = \frac{1}{h}\sum_{i=1}^{h} g_i(y_i^n)$$

Where g_i is the sum of any i where $y_i^n = y_j^n$. Since the probability of getting a consensus y_i is $p^{\mathbf{b}}(1-p)^{\mathbf{a}}$, we can order these consensuses so that y_i^n has the same ordering as Ω_n, then the expected value of the above equation is:

$$E(\hat{\zeta}_n) = \frac{1}{h}\sum_{i=1}^{h} h(p^{n-j})((1-p)^j)(y_i^n)$$

For example, if we sample h members from the population and put them in focus groups, then we should expect to create focus groups which have **a** dissenters and **b** non-dissenters $h(p^{n-j})((1-p)^j)$ times. Thus, we can write the above expected value function as:

$$E(\hat{\zeta}_n) = \sum_{i=1}^{h}(p^{n-j})((1-p)^j)(y_i^n)$$

Which is equivalent to ζ_n. Thus:

$$E(\hat{\zeta}_n) - \zeta_n = 0$$

In the next section, I will show that if we drop the homogenous focus groups X_β and X_α we tend to get better estimates than an estimate which uses both homogenous focus groups and heterogeneous focus groups.

5 Heterogeneous Estimators

Suppose, that instead of using ζ_n to estimate the final population consensus, we used an estimator that picked only the heterogeneous focus groups. Let

η_n denote a focus group estimator that drops both the homogenous non-dissenter focus group and the homogenous dissenter focus group. Thus if $\Omega_g = \Omega_n \ / \ \{X_\beta, X_\alpha\}$:

$$\eta_n = \sum_{j=1}^{n-1} \sum_{i \in \Omega_g} (p^{n-j})((1-p)^j)(x_i^n)$$

Theorem 8 and 9 will highlight two important ideas: That heterogeneous focus groups are a better estimator of population behavior when n is sufficiently large enough (Theorem 6) and when there is a large difference in opinion strength (Theorem 7). In order to further drive these theorems, I will also do the same proofs, but with an estimator that weights the all non-dissenter focus group with p^{2n} instead of p^n. In other words, I will weight the homogenous groups less than the heterogeneous groups.

5.0.3 Theorem 6

If $\eta_n \geq \mu_f$, then η_n is a better predictor than ζ_n

Proof:

We wish to show that

$$|\zeta_n - \mu_f| \geq |\eta_n - \mu_f|$$

Since we assumed that $\eta_n \geq \mu_f$, and $\zeta_n \geq \mu_f$, then

$$\zeta_n - \mu_f \geq \eta_n - \mu_f$$

$$\zeta_n \geq \eta_n$$

$$p^n \geq 0$$

Note that if we replace η_n with an estimator that weights the all non-dissenter group with a lower weight(such as p^{2n}), then the above reduces to:

$$p^n \geq p^{2n}$$

This Theorem is important because η_n starts off smaller than μ_f, but quickly grows to be larger than μ_f. This behavior can be seen below:

5.0.4 Theorem 7

If a is arbitrary large, then η_n is a better predictor of μ_f than ζ_n

Remember that from Theorem 3, $\lim_{a\to\infty} \zeta_n = p^n$. This is because both homogenous focus groups, X_α and X_β, are unaffected by changes in a or b, and thus only the consensus of the all non-dissenter group is left. Since this group is removed from η_n, then $\lim_{a\to\infty} \eta_n = 0$. As a goes to infinity the population consensus will go to zero, or $\lim_{a\to\infty} \mu_f = 0$. This means that as a goes to infinity, the heterogeneous focus group estimator will equal the population consensus, or $\lim_{a\to\infty} \eta_n = \lim_{a\to\infty} \mu_f$, while the focus group estimator ζ_n will be larger than the population consensus, or $\lim_{a\to\infty} \zeta_n \geq \lim_{a\to\infty} \mu_f$. Since the homogenous focus groups are unaffected by changes in opinion strength, $\zeta_n'(a) = \eta_n'(a)$. What this means is that the driving force behind the bias of ζ_n is that homogenous focuses groups are unaffected by changes in opinion strength and are not swayed by the opinions of others.

6 Conclusion

In the first section of this paper, I presented a model of a population which contain members that were proned to being influenced by the stronger opinions of others. Then, I constructed model of focus groups which contained members of this population. From there I showed if you weighed the results of these focus groups correctly, that this estimated how the population would behave.

This estimator approaches the behavior the population, but only because of the heterogeneous focus groups. As we increased the size and number of these focus groups, the estimator accounted for more heterogeneous interactions, thus the estimator became a more accurate picture of what would happen in the population if the policy were to take place.

A major flaw in this model is the assumption that every member in the population is just as likely to talk to another member in the population. If groups within a population are highly segregated, homogenous focus groups may be a better choice. However, the most frequent separation between focus group participants is by gender (Fern 35) [3], and men and women interact daily with each other. Would it really be wise in this case to separate focus groups by gender? Groups of people are not just people in groups, and that if focus group researchers want to understand how a group will react to a change, then they should treat focuses groups not as individuals but as a singular object.

7 Appendices

7.1 Appendix A

7.1.1 Appendix A.1

When using Bayesian inference, it is convenient to use conjugate priors, for both the population and the focus group models this paper uses the normal-normal prior conjugate relationship, where the standard deviation of the likelihood, σ^2, is known. If the prior distribution is $N(\mu_0, \sigma_0^2)$ and $x_1, \ldots, x_n$ are the signals received, then the posterior parameters are given by

$$\mu_1 = \frac{\frac{\mu_0}{\sigma_0^2} + \frac{\sum_i^n x_i}{\sigma^2}}{\frac{1}{\sigma_0^2} + \frac{n}{\sigma^2}} \quad \text{and} \quad \sigma_1^2 = (\tfrac{1}{\sigma_0^2} + \tfrac{n}{\sigma^2})^{-1} \tag{A.1.1}$$

If we let $\tau_0 = \frac{1}{\sigma_0^2}$, $n = 1$, and $\sigma^2 = 1$, then the above equations reduces to

$$\mu_1 = \tfrac{\mu_0 \tau_0 + x_1}{\tau_0 + 1} \quad \text{and} \quad \sigma_1^2 = (\tau_0 + 1)^{-1}$$

Since in the model, several iterations of the above equation occur, it may be useful to find a way to write the posterior of τ at time t, tau_t, using just tau_0. Let $\tau_t = \frac{1}{\sigma_t^2}$, thus:

$$\sigma_1^2 = (\tau_0 + 1)^{-1}$$
$$\tau_1^{-1} = (\tau_0 + 1)^{-1}$$
$$\tau_1 = \tau_0 + 1$$
$$\tau_2 = \tau_1 + 1 = \tau_0 + 2$$
$$\tau_t = \tau_0 + t$$

Thus, if x_t is the signal at time period t, then:

$$\mu_{t+1} = \frac{\mu_t(\tau_t)+x_t}{\tau_t+1}$$
$$\mu_{t+1} = \frac{\mu_t(\tau_0+t)+x_t}{\tau_0+t+1}$$

Thus, we have a similar updating function if $\tau_0 \in \{a, b\}$.

7.1.2 Appendix A.2

Referencing equation (A.1.1) from above,

$$\mu_1 = \frac{\frac{\mu_0}{\sigma_0^2}+\frac{\sum_i^n x_i}{\sigma^2}}{\frac{1}{\sigma_0^2}+\frac{n}{\sigma^2}} \quad \text{and} \quad \sigma_1^2 = (\tfrac{1}{\sigma_0^2}+\tfrac{n}{\sigma^2})^{-1}$$

Let $\pi_0 = \frac{\sigma^2}{\sigma_0^2}$, and since in the model consumers update every round, let $\sum_i^n x_i = x_1$ and $n = 1$. Thus, we can rewrite the above equation as:

$$\mu_1 = \frac{\mu_0\pi_0+x_1}{\pi_0+1} \quad \text{and} \quad \pi_1 = \frac{\pi_0}{\pi_0+1}$$

When the set of equations above are reframed this way, we can see that the likelihood takes on a new meaning with respect to opinion strength. The standard deviation of the likelihood could be thought of as the "trustworthiness" of the sender. For example, if the standard deviation of the likelihood, σ^2 is high, then x_1 is weighted less. This idea could be used for an extension or generalization of the model, but for simplicity I will assume that everyone trusts each other equally.

7.2 Appendix B

Lemma 1 *$B_t \geq A_t$ for all time periods t*

Base Case: Since the initial value of all non-dissenters is 1, then $B_0 = 1$. Likewise, $A_0 = 0$. Thus $B_0 \geq A_0$.

Inductive Step: Suppose $B_t \geq A_t$ and $a > b$. We wish to show that $B_{t+1} \geq A_{t+1}$.

Since $B_{t+1} = \frac{B_t(b+t)+E_t}{b+t+1}$ and $A_{t+1} = \frac{A_t(a+t)+E_t}{a+t+1}$:

$$\frac{B_t(b+t)+E_t}{b+t+1} \geq \frac{A_t(a+t)+E_t}{a+t+1}$$

$$(B_t(b+t) + E_t)(a+t+1) \geq (A_t(a+t) + E_t)(b+t+1)$$

$$B_t(b+t)(a+t+1) + E_t(a+t+1) \geq A_t(a+t)(b+t+1) + E_t(b+t+1)$$

$$B_t(b+t)(a+t+1) + E_t a \geq A_t(a+t)(b+t+1) + E_t b$$

$$B_t(ba + bt + b + t^2 + t + at) + E_t a \geq A_t(ba + bt + a + t^2 + t + at) + E_t b$$

$$B_t at + B_t b \geq A_t bt + A_t a$$

$$B_t(at + b) \geq A_t(bt + a)$$

Since $B_t \geq A_t$, we can simplify the above expression to:

$at + b \geq bt + a$

Which simplifies to $a \geq b$, which was assumed.

Thus, by induction $B_t \geq A_t$ for all time periods t.

Corollary 1.1 $B_t \geq E_t \geq A_t$ *for all time periods* t

First I will show that $E_t \geq A_t$, since $E_t = (1-p)A_t + (p)B_t$:

$$(1-p)A_t + (p)B_t \geq A_t$$

$$pB_t \geq A_t - A_t + pA_t$$

$$B_t \geq A_t$$

Which is true from the above proof. Next, I will show that $B_t \geq E_t$:

$$B_t \geq (1-p)A_t + (p)B_t$$

$$B_t - pB_t \geq (1-p)A_t$$

$$B_t \geq A_t$$

Which was shown in the above proof.

Thus $B_t \geq E_t \geq A_t$ for all time periods t.

Corollary 1.2 $B_{t+1} \leq B_t$ *for all time periods t*

We wish to show that $B_{t+1} \leq B_t$, since $B_{t+1} = \frac{B_t(b+t)+E_t}{b+t+1}$:

$$\frac{B_t(b+t)+E_t}{b+t+1} \leq B_t$$

$$\frac{B_t(b+t)+(1-p)A_t+(p)B_t}{b+t+1} \leq B_t$$

$$\frac{(b+t)+(1-p)\frac{A_t}{B_t}+p}{b+t+1} \leq 1$$

$$(b+t)+(1-p)\frac{A_t}{B_t}+p \leq b+t+1$$

$$(1-p)\frac{A_t}{B_t}+p \leq 1$$

$$(1-p)\frac{A_t}{B_t} \leq 1-p$$

$$\frac{A_t}{B_t} \leq 1$$

$$A_t \leq B_t$$

Which was shown in the proof above. Thus $B_{t+1} \leq B_t$ for all time periods t

Corollary 1.3 $A_{t+1} \geq A_t$ *for all time periods t*

Much like above:

$$\frac{A_t(a+t)+E_t}{a+t+1} \geq A_t$$

$$\frac{A_t(a+t)+(1-p)A_t+(p)B_t}{a+t+1} \geq A_t$$

$$\frac{(a+t)+(1-p)+(p)\frac{B_t}{A_t}}{a+t+1} \geq 1$$

$$(a+t)+(1-p)+(p)\frac{B_t}{A_t} \geq a+t+1$$

$$-p + (p)\frac{B_t}{A_t} \geq 0$$

$$B_t \geq A_t$$

Which was shown in the proof above. Thus $A_{t+1} \geq A_t$ for all time periods t.

7.3 Appendix C

7.3.1 Lemma 2

Suppose at round r_u, $max(X_n) = \gamma_{k,r}$ and $min(X_n) = \gamma_{l,r}$, and $\gamma_{k,r} \neq \gamma_{l,r}$, then:

$\gamma_{l,r} < \gamma_{l,r+1}$ and $\gamma_{k,r+1} < \gamma_{k,r}$

Proof:

We can rewrite $\gamma_{k,r+1}$ as:

$$\gamma_{k,r_u+1} = \frac{\gamma_{k,r_u}(c + r_u) + \sum_{j=1,j\neq i}^{n} \gamma_{j,r_u-j+1}}{c + r_u + 1} \tag{9}$$

Let the mean of all γ_i on round r_u be denoted by $\bar{\gamma}_{r_u}$. Thus, we can rewrite the summation as

$$\gamma_{k,r_u+1} = \frac{\gamma_{k,r_u}(c+r_u)+(n)\bar{\gamma}_{r_u}-\gamma_{k,r_u}}{c+r_u+1}$$

$$\gamma_{k,r_u+1} = \frac{\gamma_{k,r_u}(c+r_u-1)+(n)\bar{\gamma}_{r_u}}{c+r_{u+1}}$$

Thus, we wish to show:

$$\frac{\gamma_{k,r_u}(c+r_u-1)+(n)\bar{\gamma}_{r_u}}{c+r_{u+1}} < \gamma_{k,r_u}$$

Since $r_u = u(n-1)$:

$$\frac{\gamma_{k,r_u}(c+u(n-1)-1)+(n)\bar{\gamma}_{r_u}}{c+(u+1)(n-1)} < \gamma_{k,r_u}$$
$$\gamma_{k,r_u}(c+u(n-1)-1)+(n)\bar{\gamma}_{r_u} < \gamma_{k,r_u}(c+(u+1)(n-1))$$
$$(n)\bar{\gamma}_{r_u} < \gamma_{k,r_u}[(c+(u+1)(n-1))-(c+u(n-1)-1)]$$
$$(n)\bar{\gamma}_{r_u} < (n)\gamma_{k,r_u}$$
$$\bar{\gamma}_{r_u} < \gamma_{k,r_u}$$

Since γ_{k,r_u} is the max at round r_u, this is true. A symmetric proof can be done to show that $\gamma_{l,r} \leq \gamma_{l,r+1}$. Thus, for any round r_u with $max(X_n) = \gamma_{k,r}$ and $min(X_n) = \gamma_{l,r}$, $\gamma_{l,r} < \gamma_{l,r+1} < \gamma_{k,r+1} < \gamma_{k,r}$.

7.3.2 Lemma 3

Suppose $\beta_{j,r_u} < \beta_{i,r_u}$, then $\beta_{j,r_{u+1}} < \beta_{i,r_{u+1}}$.

Proof:

This follows trivially from the generalized updating formula:

$$\frac{\beta_{j,r_u}(b+r_u-1)+n\bar{\gamma}_{r_u}}{b+r_{u+1}} < \frac{\beta_{i,r_u}(b+r_u-1)+n\bar{\gamma}_{r_u}}{b+r_{u+1}} \tag{10}$$

Likewise, if $\alpha_{j,r_u} \leq \alpha_{i,r_u}$, then $\alpha_{j,r_{u+1}} \leq \alpha_{i,r_{u+1}}$.

7.3.3 Corollary 3.1

For each focus group $X_{i,n}$, there is a unique α_i and β_j so that at the end of any round r_u, $min(X_{i,n}) = \alpha_i$ and $max(X_{i,n}) = \beta_j$. That is, there is a unique max and min for each focus group.

7.3.4 Lemma 5

For any focus group X_i^n, $x_i^{n\prime}(a) \leq 0$.

Proof:

We know from Corollary 3.1, that for any X_i^n, there exists a element α_{i,r_u} which is always the minimum for any u. Thus, if $a_z \geq a_s$, we wish to show:

$$\frac{\alpha_{i,r_u}(a_z+r_u-1)+n\bar{\gamma}_{r_u}}{a_z+r_{u+1}} \leq \frac{\alpha_{i,r_u}(a_s+r_u-1)+n\bar{\gamma}_{r_u}}{a_s+r_{u+1}}$$

$$\alpha_{i,r_u}(a_z+r_u-1)(a_s+r_{u+1})+n\bar{\gamma}_{r_u}(a_s+r_{u+1}) \leq$$

$$\alpha_{i,r_u}(a_s+r_u-1)(a_z+r_{u+1})+n\bar{\gamma}_{r_u}(a_z+r_{u+1})$$

$$a_s[(r_u-1)\alpha_{i,r_u}+n\bar{\gamma}_{r_u}] \leq a_z[(r_u-1)\alpha_{i,r_u}+n\bar{\gamma}_{r_u}]$$

$$a_s \leq a_z$$

Thus $x_i^{n\prime}(a) \leq 0$.

7.3.5 Lemma 6

$\lim_{a\to\infty} x_i^n = 0$

Proof:

$$\alpha_{k+1,r_{u+1}} = \frac{\alpha_{k,r_u}(a+r_u-1)+n\bar{\gamma}_{r_u}}{a+r_{u+1}}$$

$$\lim_{a\to\infty} \frac{\alpha_{k,r_u}(a+r_u-1)+n\bar{\gamma}_{r_u}}{a+r_{u+1}}$$

$$\lim_{a\to\infty} \frac{\alpha_{k,r_u}(a+r_u-1)}{a+r_{u+1}} + \lim_{a\to\infty} \frac{n\bar{\gamma}_{r_u}}{a+r_{u+1}}$$

$$(\alpha_{k,r_u}) \lim_{a\to\infty} \frac{(a+r_u-1)}{a+r_{u+1}}$$

$$(\alpha_{k,r_u})(1)$$

$$\alpha_{k+1,r_{u+1}} = \alpha_{k,r_u}$$

$$\alpha_{k+1,r_{u+1}} = \alpha_{k,r_u} = \alpha_{k-1,r_u} = \cdots = \alpha_{0,r_u} = 0$$

Since $lim(\alpha_i)$ is the consensus, we can conclude that $\lim_{a\to\infty} x_i^n = 0$.

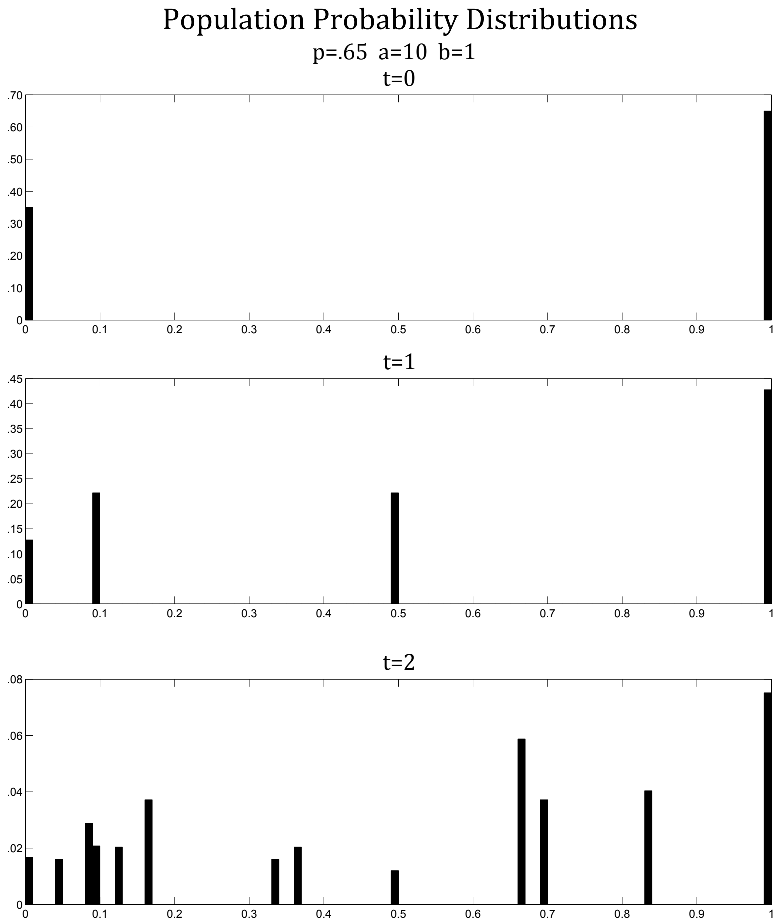
7.4 Appendix D

At the end of the first round without including the last member, the set of the last members of each focus group, ω_n, is equivalent to set of values(or members) at time $n = t$ in the expected value population model, E_t.

7.4.1 Example: Focus Groups of size two

Suppose n=2, then we have the focus groups $\Omega_2 = \{\beta, \beta\}, \{\alpha, \beta\}, \{\beta, \alpha\}, \{\alpha, \alpha\}$. If we only allow the first members to send signals, then the last members have posteriors $\{\beta|\beta\}, \{\beta|\alpha\}, \{\alpha|\beta\}, \{\alpha|\alpha\}$. At time two in the population model, the possible types of posterior distributions are $\{\beta|\beta\}, \{\beta|\alpha\}, \{\alpha|\beta\}, \{\alpha|\alpha\}$.

7.4.2 Appendix D.1



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